PHYSICS NYB-07 H06

Lecture 8: Using and visualizing electric potential

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Review

- A point charge q creates an electric field $\vec{E} = k_e \frac{q}{r^2}\hat{r}$
- A charge q_0 placed in an electric field $\vec{E}$ feels a force $\vec{F}_e = q_0\vec{E}$
- This force does work on the charge as it moves from point A to point B
$$W_e = \int_A^B \vec{F}_e \cdot d\vec{s} = q_0 \int_A^B \vec{E} \cdot d\vec{s}$$
- The work done by the force is equal to minus the difference in the electric potential energies at points A and B, $\Delta U_e = -W_e = -q_0 \int_A^B \vec{E} \cdot d\vec{s} = U_B - U_A$

Review

- The electric potential energy for two charges separated by a distance r is $U(r) = k_e \dfrac{q_0 q}{r}$
- Energy is conserved, $E_{tot} = K + U =$ cst
- A charge q has an electric potential associated with it $V(r) = k_e \dfrac{q}{r}$
- When a charge q_0 is placed at a location where the electric potential is V, the resultant potential energy is $U_e = q_0 V$
- The components of the electric field are related to the potential through $E_r = -\dfrac{\partial}{\partial r} V(r)$

Examples

Let's check this relation for the electric field of a point charge.

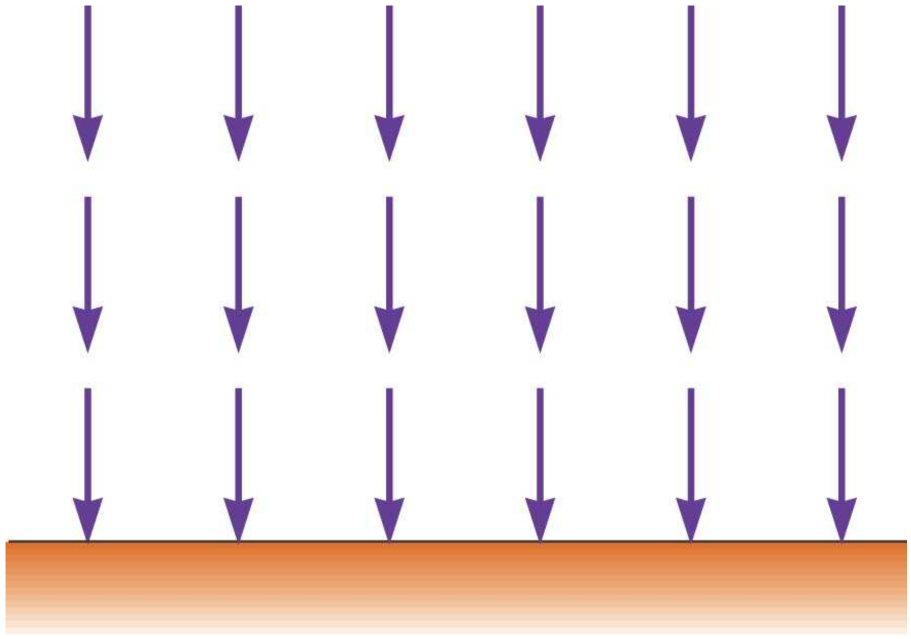
We know that the electric potential created by a point charge is $V(r) = k_e \frac{q}{r}$. According to what we've just said,

$$\begin{aligned} E_r &= -\frac{\partial}{\partial r} V(r) = -\frac{\partial}{\partial r}\left(k_e \frac{q}{r}\right) = -k_e q \left(-\frac{1}{r^2}\right) \\ \Rightarrow E_r &= k_e \frac{q}{r^2} \end{aligned}$$

as it should be. Notice that taking the derivative with respect to other coordinates gives zero since $V(r)$ only depends on r, so this means that $\vec{E} = k_e \frac{q}{r^2}\hat{r}$, again, exactly as it should.

Examples

What is the electric potential when we have a uniform electric field pointing in the negative y-direction?



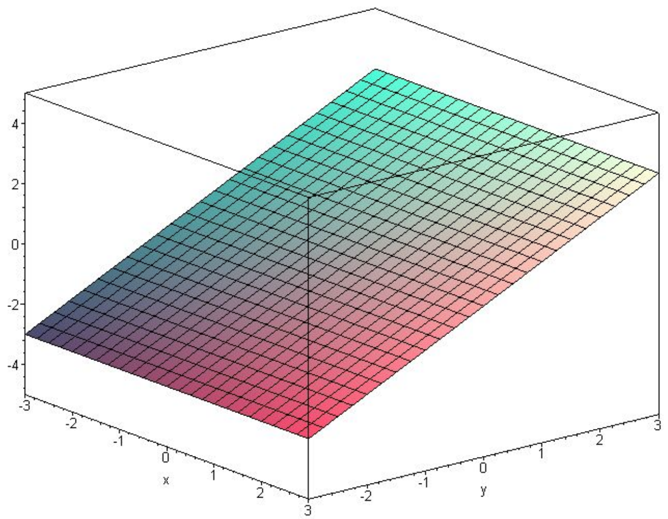
(b)

Examples

The electric field has the form $\vec{E} = -C\hat{j}$, so we know we are looking for a potential that depends only on y, or else the electric field would have other components. We have

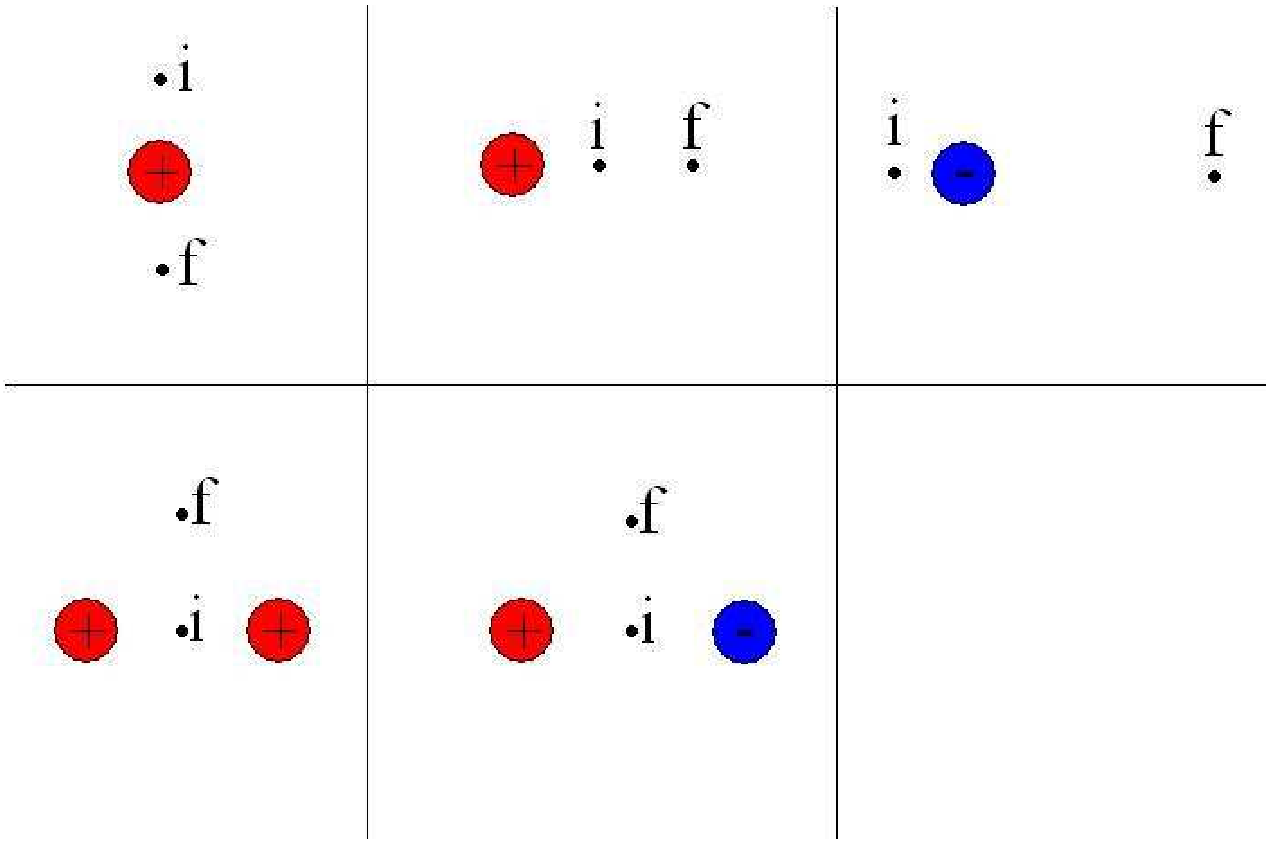
$$E_y = -\frac{\partial}{\partial y}V(y) = -C$$

So we need a potential function whose derivative with respect to y is a constant. This is easy, $V(y) = Cy$. Graphically, this looks like this



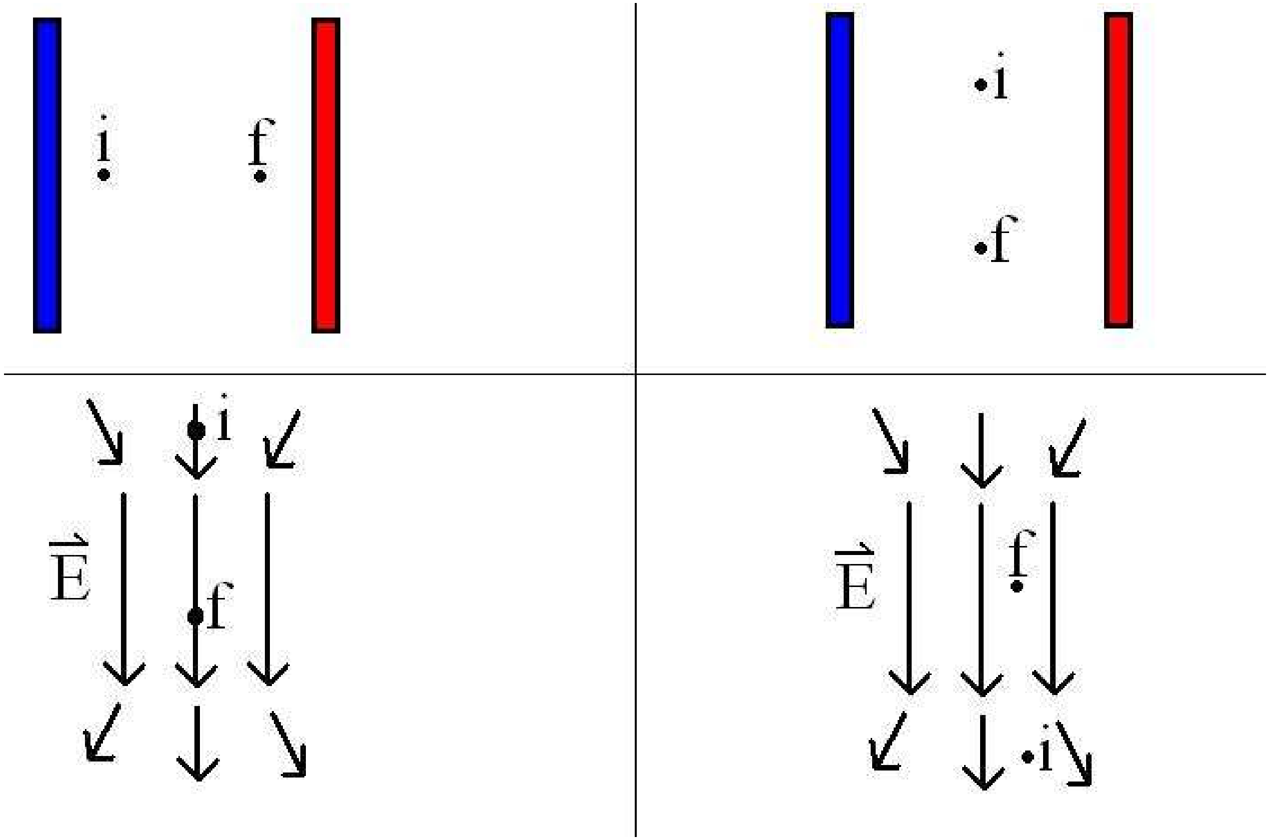
Examples

Is ΔU positive, negative or zero as a proton moves from i to f in the diagrams below? What about an electron? A hydrogen atom? Repeat, but for ΔV.

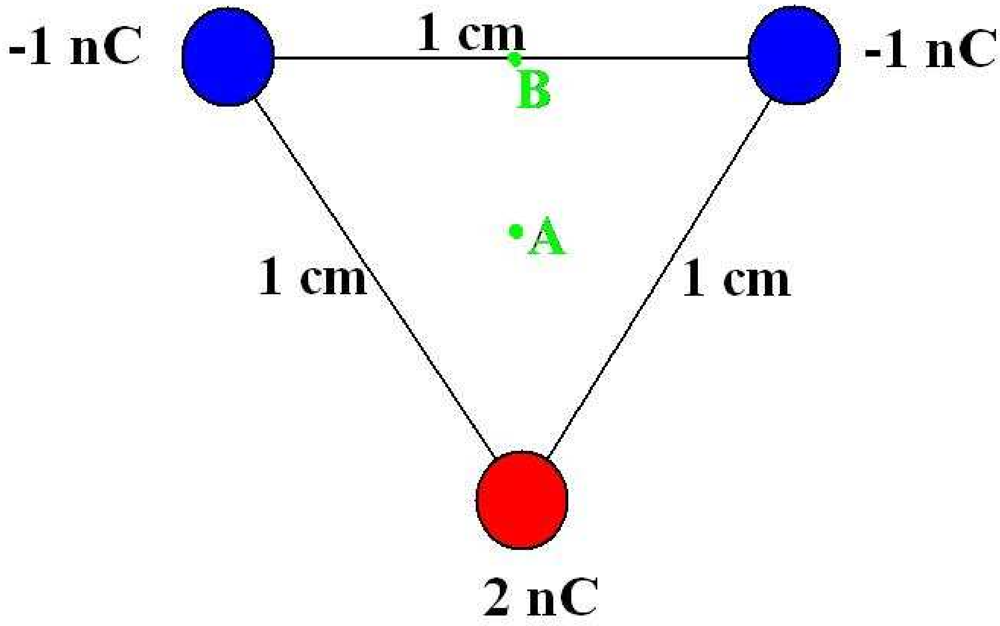


Examples

Is ΔU positive, negative or zero as a proton moves from i to f in the diagrams below? What about an electron? A hydrogen atom? Repeat, but for ΔV.



Review



What is the potential difference between point A and point B. How much work is done by the electric field on an electron that goes from point A to point B? On a proton?

Review

The potential at point A is $V = k_e \frac{q_1}{r_1} + k_e \frac{q_2}{r_2} + k_e \frac{q_3}{r_3}$ where $r_1 = r_2 = r_3$ and $q_1 + q_2 + q_3 = 0$, so at point A we have $V_A = 0$. At point B, we have two -1 nC charge 0.5 cm away, and one 2 nC charge $\sqrt{(0.01)^2 - (0.005)^2}$ m away. This leads to a potential of $V_B =$

$$8.99 \times 10^9 \left(2 \frac{-1 \times 10^{-9}}{0.005} + \frac{2 \times 10^{-9}}{\sqrt{(0.01)^2 - (0.005)^2}} \right) = -1520 \text{ V}.$$

So the potential difference between points B and A is $\Delta V = V_B - V_A = -1520$ V. The change in the potential energy of a charge moved from A to B is $\Delta U = q\Delta V$. For an electron, the work done by the electric field is $W_e = -\Delta U = -(-1.6 \times 10^{-19} \times (-1520)) = -2.432 \times 10^{-16}$ J. For a proton, it is $W_e = +2.432 \times 10^{-16}$ J.

Review

A single electron is orbiting a single proton at a radius $r_0 = 0.0529$ nm (you should recognize what element this is, and what state it is in!). Find the speed of the electron, knowing that its total energy is -13.6 eV. How much work would need to be done in order to bring this electron infinitely far away from the proton?

The total energy is $E_{tot} = K + U = \frac{1}{2}m_e v^2 + (-e)V$, where V is the electric potential a distance r_0 from the proton, $V = k_e \frac{(+e)}{r_0}$. This means that $\frac{1}{2}m_e v^2 = E_{tot} + k_e \frac{e^2}{r_0}$. Remembering that 1 eV $= 1.6 \times 10^{-19}$ J, and solving for the speed, we find $v = 2.19 \times 10^6$ m/s. When the electron is infinitely far away and at rest, the total energy is 0, so we need to add 13.6 eV to the system.

Equipotentials

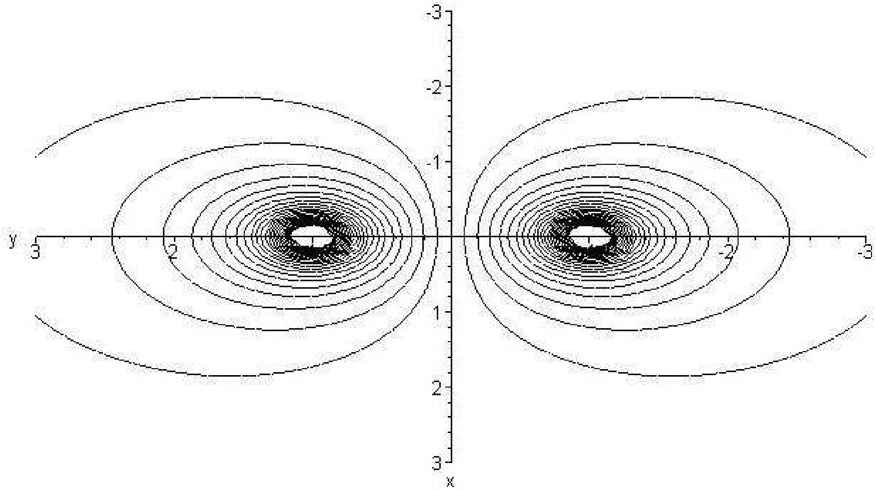
Definition:

An *equipotential* is a surface along which the potential remains constant. The potential energy of a charge moving along an equipotential is therefore constant, which means that no work is done by the electric field on a charge moving along an equipotential.

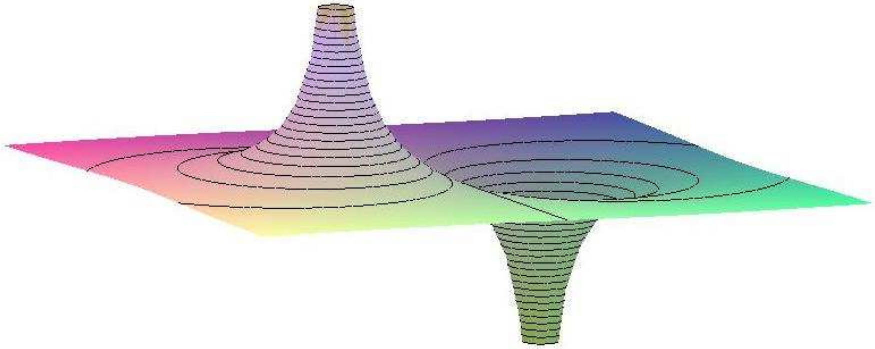
- The electric field is everywhere perpendicular to equipotentials
- The magnitude of the electric field is greater where equipotentials are more **closely spaced**

Equipotentials

Equipotentials look like this

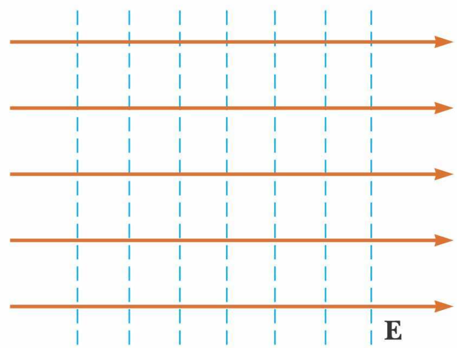


You can visualize what they represent with the surfaces.
They represent paths of *constant height* on the surfaces.



Equipotentials and field lines

Field lines and equipotentials are everywhere perpendicular to each other



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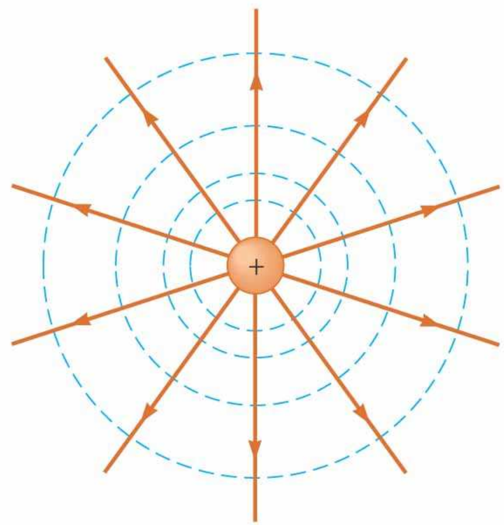

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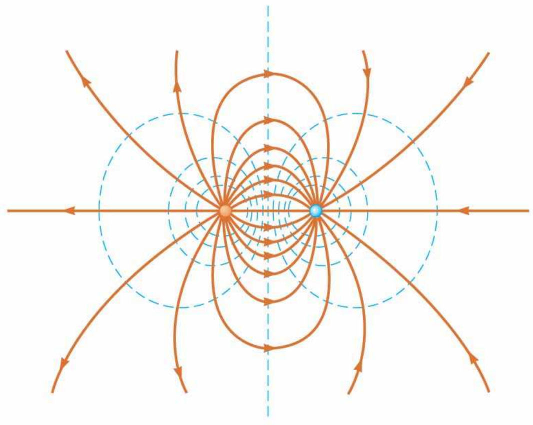
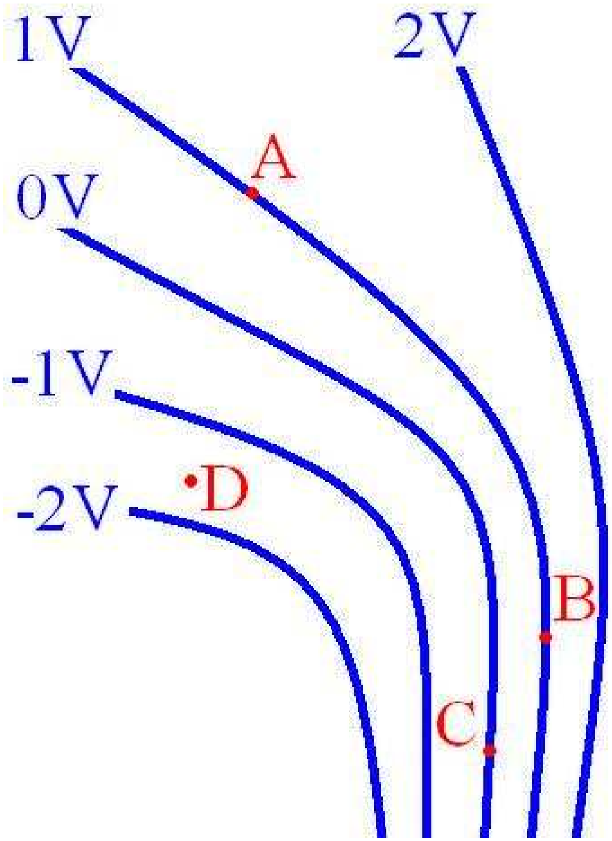


Examples

The figure below shows 5 equipotential lines.

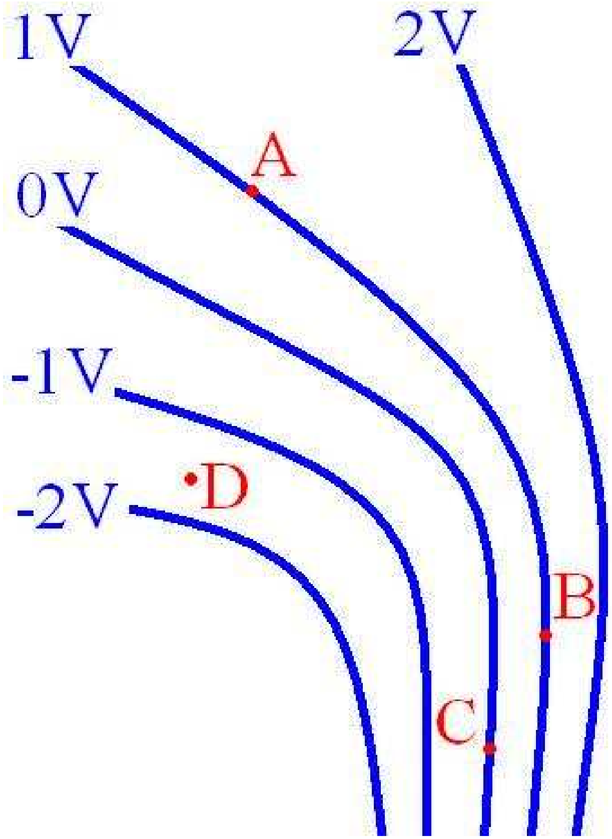


Which way is the electric field pointing at the indicated points? Decreasing V, perpendicular to the equipotential lines

Where is its magnitude greatest? Point C

Examples

The figure below shows 5 equipotential lines.

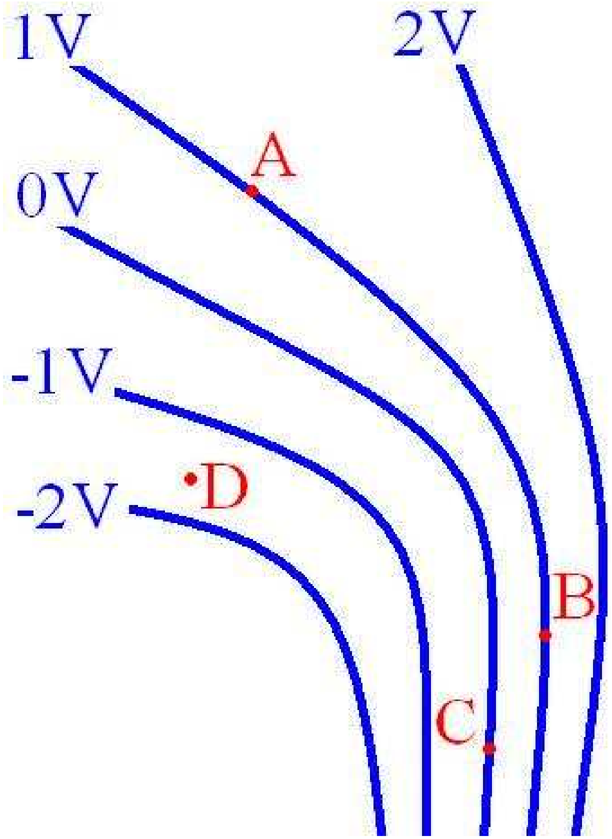


What's the change in the potential energy of a proton going from A to B? 0 eV

What's the change in the potential energy of a proton going from A to C? -1 eV

Examples

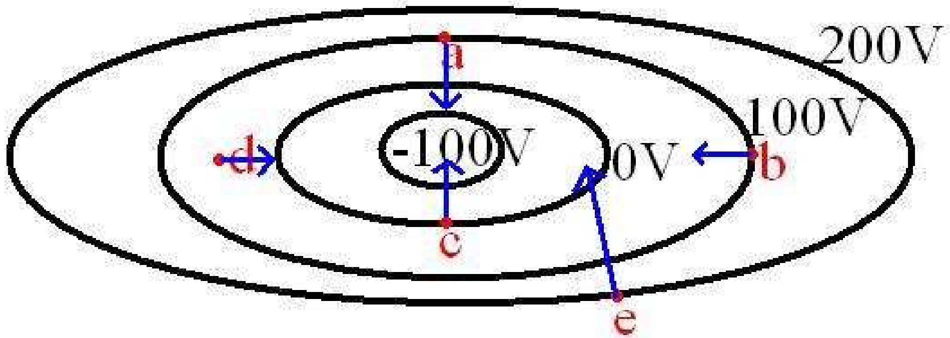
The figure below shows 5 equipotential lines.



What's the change in the potential energy of an electron going from A to B? 0 eV

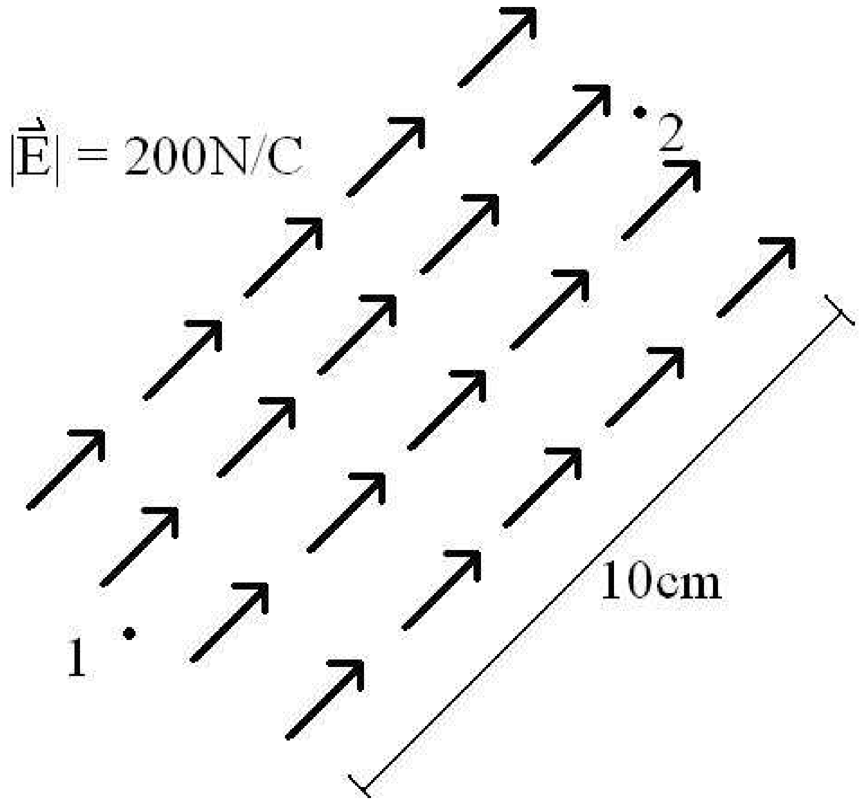
What's the change in the potential energy of an electron going from A to C? +1 eV

Examples



Compare E_a and E_b $E_a > E_b$
Compare E_c and E_d $E_c > E_d$
Draw the electric field vectors at points $a - e$

Examples

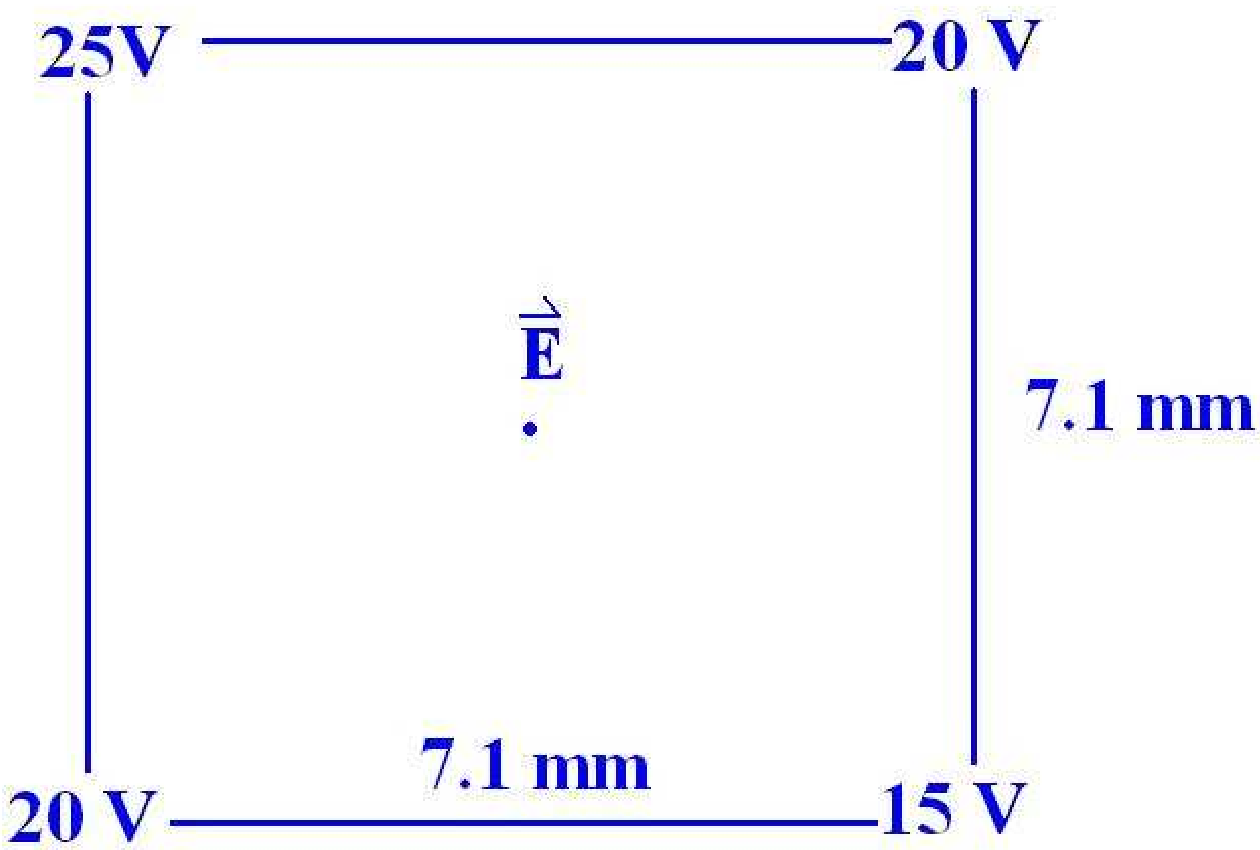


What is the potential difference between points 1 and 2?
Draw equipotential surfaces separated by $\Delta V = 5$ V

Examples

Since the electric field has a magnitude of 200 N/C and this is equivalent to 200 V/m, this means that the potential changes by a value of 20 V over a 10 cm distance. The electric field points in the direction of decreasing potential, therefore the potential is 20 V lower at point 2 than point 1. Equipotentials separated by 5 V will be separated by 2.5 cm, and will be perpendicular to the direction of the electric field vectors pictured here.

Examples



What is the magnitude and direction of the electric field at the center of the square pictured here?

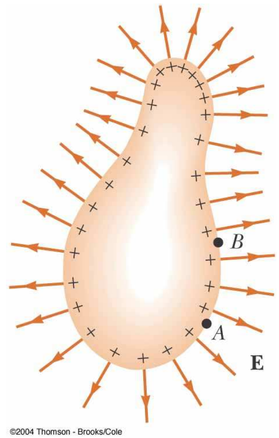
Examples

Given the configuration, it is clear the diagonal is 1 cm long. Over the diagonal, the potential decreases by 10 V over a 1 cm distance, so the electric field has magnitude of 1000 V/m. Give the configuration, it is safe to assume that at the center of the square the field is pointing towards the right bottom corner, where the potential has its smallest value.

Potential inside conductors

- We know that the electric field inside a conductor in electrostatic equilibrium vanishes
- $\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) = 0$
- So V doesn't depend on x, y or z
- V is *constant* inside a conductor in electrostatic equilibrium
- "CONSTANT" DOES NOT MEAN "ZERO"!!!

Potential inside conductors

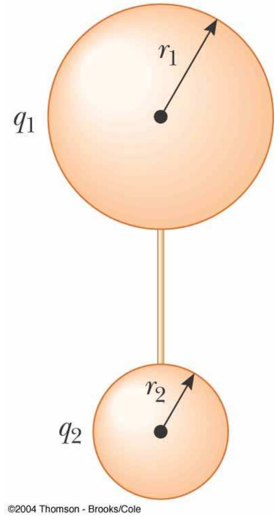


- The potential inside a conductor is constant, which means V inside is equal to whatever V is at the surface of the conductor.
- This means the whole volume of the conductor, including the surface, is an equipotential
- Remember, the electric field is perpendicular to equipotential surfaces
- So *the electric field outside a conductor is perpendicular to the surface*.

Potential inside conductors

So the electric field is perpendicular to the surface of a conductor.

Can we find out where the electric field is greater? Where the surface is highly curved? Where it is less curved?



We have what we need to figure this out.

Potential inside conductors

The potential at the surface of sphere 1 is equal to the potential at the surface of sphere 2, so we can write (remember the potential and sphere outside a spherical distribution are the same as that of a point charge in the center of the distribution; we'll prove this later on)

$$V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2}$$

from which we obtain $\frac{q_1}{q_2} = \frac{r_1}{r_2}$. The magnitude of the electric field is $E_1 = k_e \frac{q_1}{r_1^2}$, $E_2 = k_e \frac{q_2}{r_2^2}$, $\Rightarrow \frac{E_1}{E_2} = \frac{r_2}{r_1}$ so the field is greater for the smaller radius of curvature.

Example

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What's going on in this nice picture? What is the charge and radius of the Van de Graaf pictured here if it sparks when we bring it up to a potential of 600 kV?

Example

When the electric field at the surface of the Van de Graaf generator is greater than the surrounding air's *dielectric strength*, 3.00×10^6 V/m, electrons accelerated in the field knock of electrons in the surrounding atoms, which then accelerate in the field and knock off electrons in the surrounding atoms, and so on... (This is exactly the same phenomenon that we saw with lightning). The magnitude of the electric field at the surface of a charged metal sphere is $E = k_e \frac{q}{r^2}$ while the potential is $V = k_e \frac{q}{r}$. Therefore, $r = \frac{V}{E}$, and since E must be 3.00×10^6 V/m, and $V = 600$ kV, we have $r = 0.2$ m. Plugging this into either E or V gives us $q = 13.3\ \mu$C.

Assignment 3

- Chapter 25, problems 8, 9, 12, 18, 19, 28, 32, 39, 40

What to read for next lecture

- 27.1, 27.2